Interaction of ultrarelativistic electron and proton bunches with dense plasmas

A. A. Rukhadze[a], S. P. Sadykova [*b]

[a]*Prokhorov General Physics Institute, Russian Academy of Sciences 119991, Vavilov Str., 38., Moscow, Russia* [†]
[b]*Humboldt-Universität zu Berlin, Germany, Newtonstr. 15, 12489 Berlin, Germany*

Abstract

Here we discuss the possibility of employment of ultrarelativistic electron and proton bunches for generation of high plasma wakefields in dense plasmas due to the Cherenkov resonance plasma-bunch interaction. We estimate the maximum amplitude of such a wake and minimum system length at which the maximum amplitude can be generated at the given bunch parameters.

Keywords:
Electron-bunch-driven, proton-bunch-driven plasma wakefield acceleration, Ultrarelativistic bunch, Cherenkov resonance, Plasma-bunch interaction

1. Introduction

For the first time in works [1, 2] it was proposed to employ the relativistic electron bunches propogating through plasma for generation of high plasma wakefields. High-energy bunch electrons generate the wake in such a way that the energy from a bunch of electrons is transferred to the plasma wave through the Cherenkov resonance radiation producing high electric fields (wakefields). Later, in the work [3] it was shown that the ultrarelativistic electron bunches can generate plasma waves with high relativistic phase velocity including the transverse electromagnetic waves efficiently radiated out of plasma. This led to the birth of a new applied science - high-power pulse

[*][Corresponding author]: Humboldt-Universität zu Berlin, Germany, Newtonstr. 15, 12489 Berlin, Germany; Tel.: +493020937876, Fax.: +49-30-2093-7638; *E-mail adress*: saltanat@physik.hu-berlin.de.

[†]*E-mail adress*: rukh@fpl.gpi.ru .

relativistic superhigh-frequency (SHF) electronics [4],[5] enabling to produce the high-power pulse broadened SHF amplifiers and generators on a base of such plasma-bunch systems [6].

Recently, in works [7, 8] the possibility of generation of high power wakefields (proton-bunch-driven plasma-wakefield acceleration) of terra-watt amplitude using the ultrarelativistic proton bunches was introduced.

In the present work this idea along with the employment of electron bunch is discussed at the qualitative level. Namely, we make an estimation of plasma parameters, maximum amplitude of the generated wake when the relativistic electron and proton bunches are used and sysem length at which the maximum amplitude of the wake can be gained.

2. Ultrarelativistic electron bunch

Let us start our analysis with the ultrarelativistic electron bunch of density n_b and velocity $\vec{u}$, noting that

$$\gamma = \frac{1}{\sqrt{(1 - u^2/c^2)}} >> 1. \tag{1}$$

Such a bunch interacts with the plasma of density $n_p >> n_b$ and generates the plane wave $E = E_0 \exp(-i\omega t + i\vec{k}\cdot\vec{r})$. We choose an axis Z directed along the velocity of the bunch $\vec{u}$, and put the external field as absent [1], disperse relation describing the amplification of a plasma wave with the help of a bunch can be written as following [3] :

$$(k^2c^2 - \omega^2 + \omega_p^2 + \omega_b^2\gamma^{-1})\left(1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2\gamma^{-3}}{(\omega - k_z u)^2}\right) - \frac{k_\perp^2 u^2}{\omega^2}\frac{\omega_p^2\omega_b^2\gamma^{-1}}{(\omega - k_z u)^2} = 0, \tag{2}$$

here $\omega_p = \sqrt{4\pi e^2 n_p/m}$ - Langmuir plasma electron frequency, k_z and $k_\perp$ are the logitudinal (directed along the velocity of the bunch $\vec{u}$) and transverse components of the wave vector $\vec{k}$.

The general solution of the equation (2) $\omega(\vec{k})$ with the positive imaginary

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[1] In ultrarelativistic bunches bunch divergence can be neglected because when the electron bunch is ejected into the dense plasma, within the time $t \sim 1/\omega_p$ the neutralization of the bucnh charge occurs prohibitting the bunch divergence [4].

part ($\Im m\omega > 0$)

$$\omega = k_z u(1+\delta) = \omega_p(1+\delta)$$

$$\delta = \frac{-1+i\sqrt{3}}{2}(\frac{n_b}{2n_p}\frac{1}{\gamma})^{1/3}(1 - \frac{u^2}{c^2}\frac{\omega_p^2}{\omega_p^2 + k_\perp^2 u^2})^{1/3}. \tag{3}$$

Since the value δ (increment), determining the temporary growth of the plasma wave amplitude generated by the bunch, depends on $k_\perp$, then in this way it depends on the transverse size $k_\perp$ of the considered plasma-bunch system. For a start we would like to be far away from the concrete magnitudes of the concrete system parameters and consider two opposite limitting cases:

A . In a case of dense plasma when

$$\omega_p^2 >> k_\perp^2 u^2 \tag{4}$$

Then, the value δ is given by the expression

$$\delta = \frac{-1+i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\right)^{1/3}\frac{1}{\gamma}. \tag{5}$$

B . In a case of rare plasma when the inverse to (4) inequality is satisfied then the value δ becomes

$$\delta = \frac{-1+i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\frac{1}{\gamma}\right)^{1/3}. \tag{6}$$

Let us outline again that while deriving the equations (3-6) we took into account that value δ is negligibly small what is sustained by the inequalities $n_b << n_p$ and $\gamma^2 >> 1$.

We would like to stress that for the development of instability and plasma wave growth it is necessary that the following constraint for a system length must be satisfied:

$$L > \frac{u}{\Im m\delta\omega_p} \tag{7}$$

being at the same time limited due to the bunch divergence to the following constraint :

$$L < \frac{u\sqrt{\gamma}}{\omega_b} = \frac{u\sqrt{\gamma}}{(n_b/n_p)^{1/2}\omega_p}.$$

Let us now estimate the maximum amplitude of the generated by the bunch wake, so called the amplitude of saturated instability. For this, we choose the wake as a frame of reference moving with the speed ω/k with respect to the laboratory frame of reference (plasma in a rest). In accordance with the Lorentz transformations the speed of bunch electrons in the chosen frame will be

$$u_1 = -\frac{u\Im m\delta\gamma^2}{1 - \frac{2u^2}{c^2}\Im m\delta\gamma^2} = -\frac{u\Im m\delta\gamma^2}{1 - 2\Im m\delta(\gamma^2 - 1)}. \tag{8}$$

From Eq. (8) follows that in the relativistic limit when $(\delta\gamma^2 << 1)$ this speed is small $u_1 \approx -u\delta\gamma^2 << u$, whereas in the ultrarelativistic limit $(\delta\gamma^2 >> 1)$ the speed is high $u_1 \approx u/2$.

It is obvious that the saturation of instability can occur when the kinetic energy of electrons, in the wake frame of reference, will become less than the amplitude of the potential of the plasma wake measured in the same frame. In this case the bunch electrons get trapped by the wake, i.e. there will be no relative motion between the bunch electrons and the wake, thus, no energy exchange between the bunch and wake occurs, the bunch and the wake become stationary. Hence, the stationary saturation amplitude of the plasma wake potential can be obtained as following

$$\frac{e\Phi_0}{mc^2} = \frac{1}{\gamma}\left\{\frac{1}{\sqrt{1 - \frac{u^2\delta^2\gamma^4}{c^2(1 - 2\delta(\gamma^2 - 1))^2}}} - 1\right\}, \tag{9}$$

here $\Phi_0\gamma$ is measured in the wake frame of reference. In the relativistic limit when $(\delta\gamma^2 << 1)$ we will get the result already obtained in [9] for dense plasma Eq. (5), whereas in the ultrarelativistic limit when $(\delta\gamma^2 >> 1)$ we obtain

$$\frac{e\Phi_0}{mc^2} \approx \frac{0.154}{\gamma}, \tag{10}$$

here Φ_0 is measured in the laboratory frame of reference. Eq. (10) leads to the following conclusion: the amplitude of the potential of the plasma wake generated by the electron bunch is dependent on the relativity factor $\Phi_0 = \frac{0.154mc^2}{e\gamma}$. This is demonstrated in the Figure 1. We can obeserve a maximum $\Phi_{0max} \simeq 770$ V for dense plasma at $n_p = 10^{16}cm^{-3}$, $\gamma_0 \simeq 24$ and $\Phi_{0max} \simeq 15.5 \cdot 10^3$ V $\simeq 15.5$ k V rare plasma at $n_p = 6 \cdot 10^{12}cm^{-3}$, $\gamma_0 \simeq 2.56$, see Appendix A. It is quite easy to estimate the amplitude of the electrial

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field of the wake generated by the bunch. For this in Eq. (9) one needs to take into account that $E_0 \sim k\Phi_0$ leading to the following equation for the relative energy density of the electrostatic field

$$\frac{E_0^2}{8\pi n_p m c^2 \gamma} \simeq \frac{c^2}{u^2} \frac{1}{2\gamma^3} \left\{ \frac{1}{\sqrt{1 - \frac{u^2 \delta^2 \gamma^4}{c^2 (1 - 2\delta(\gamma^2 - 1))^2}}} - 1 \right\}^2 . \qquad (11)$$

In the relativistic limit when $(\delta\gamma^2 << 1)$ this equation turns into the result obtained in [9] for dense plasma Eq. (5), whereas in the ultrarelativistic limit when $(\delta\gamma^2 >> 1)$ we get

$$\frac{E_0^2}{8\pi n_p m c^2 \gamma} \simeq \frac{0.012}{\gamma^3} \qquad (12)$$

We can readily see that with an increase of relativity the amplitude of the electric field of the wake generated by the electron bunch in its turn is dependent on the relativity γ as it is shown in the Figure 1 and is also defined in the ultrarelativistic limit by the plasma density n_p, $E_0 \simeq 0.024\sqrt{4\pi n_p m c^2}/\gamma$. For example, for dense plasma at $\gamma_0 \simeq 24$, $n_p = 10^{16} cm^{-3}$, we have $E_{0max} \simeq 14.5 \cdot 10^6$ V/cm $\simeq 14.5$ M V/m, whereas for rare plasma at $n_p = 6 \cdot 10^{12} cm^{-3}$, $\gamma_0 \simeq 11.6$ we have $E_{0max} \simeq 7.2 \cdot 10^6$ V/cm $\simeq 7.2$ M V/m.

3. Ultrarelativistic proton bunch

As it has been mentioned above, in works [7, 8] the possibility of generation of high power wakefields (proton-bunch-driven plasma-wakefield acceleration) of terra-watt amplitude using the ultrarelativistic proton bunches of giga-electron-volt energy was introduced. The idea is that to slow the protons down using the plasma wakefield is much harder than the electrons and as a result the significantly higher wakefield amplitude can be gained in comparison with that produced by the electron bunch.

The disperse relation (2) can be easily generalized for the case of plasma-proton-bunch interaction. Correspondingly, we need to make the substitution $n_b \rightarrow n_{bi}$ (n_b - electron bunch density), i.e. substitute the n_b by the proton bunch density n_{bi} multiplied by the ratio of electron mass m to the ion mass M, m/M.

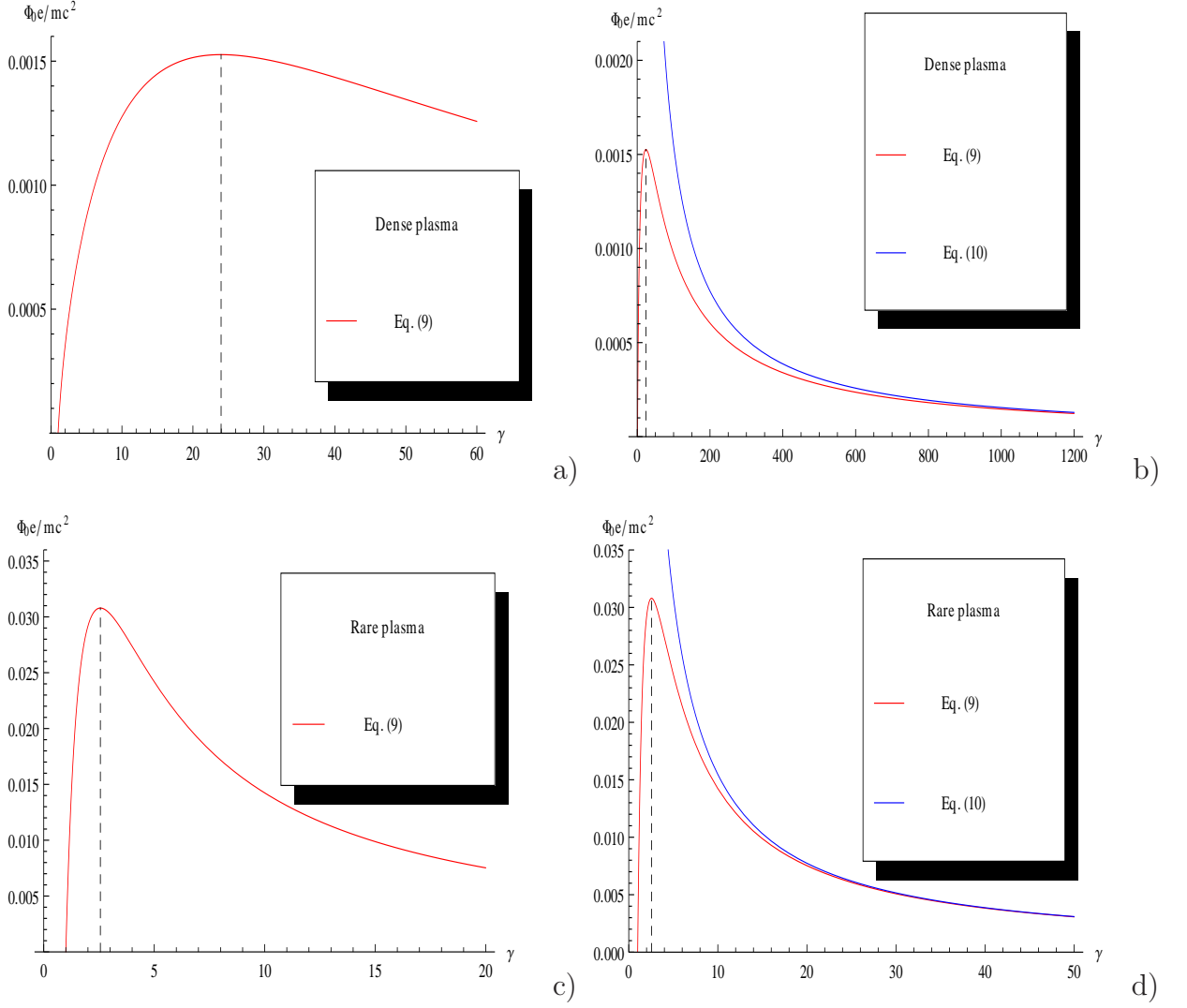Having solved Eq. (2), we write the solution generalizing Eq. (3) for the

Figure 1: a) The relative stationary saturation amplitude of the plasma wake potential generated by the electron bunch Eq. (9) for a), b) dense (5) at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 10^{16} cm^{-3}$, $\gamma_0 = 24$ and c), d) rare (6) plasmas at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 6 \cdot 10^{12} cm^{-3}$, $\gamma_0 = 2.6$. In b) and d) the relative stationary saturation amplitude in the ultrarelativistic limit (10) is presented for comparison.

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proton bunch down

$$\omega = k_z u(1 + \delta_1) = \omega_p(1 + \delta_1)$$

$$\delta_1 = \frac{-1 + i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\frac{m}{M}\frac{1}{\gamma}\right)^{1/3}\left(1 - \frac{u^2}{c^2}\frac{\omega_p^2}{\omega_p^2 + k_\perp^2 u^2}\right)^{1/3}, \qquad (13)$$

here δ_1 differs from δ, from Eq. (3), in $n_b \to n_{bi}\frac{m}{M}$.

A . In the dense plasma when the constraint (4) is satisfied (cmpr. with (5)) the value δ_1 takes the following view:

$$\delta_1 = \frac{-1 + i\sqrt{3}}{2}\left(\frac{n_{bi}}{2n_p}\frac{m}{M}\right)^{1/3}\frac{1}{\gamma}. \qquad (14)$$

B . In the rare plasma when the inverse to (4) inequality is satisfied then the value δ_1 (cmpr. with (6)) becomes

$$\delta_1 = \frac{-1 + i\sqrt{3}}{2}\left(\frac{n_{bi}}{2n_p}\frac{m}{M}\frac{1}{\gamma}\right)^{1/3}. \qquad (15)$$

We would like to point out again that as in the case for the electron bunch the development of proton bunch instability and plasma wake growth can occur when the following constraint similar to Eq. (7) for a system length must be satisfied

$$L > \frac{u}{\Im m \delta_1 \omega_p}. \qquad (16)$$

However, the system length must be limited as well in order to neglect the proton bunch divergence due to the proton charges repulsion

$$L < \frac{u\sqrt{\gamma}}{\omega_{bi}} = \frac{u\sqrt{\gamma}}{\left(\frac{n_{bi}}{n_p}\frac{m}{M}\right)^{1/2}\omega_p}.$$

Similarly, we estimate the maximum amplitude of the generated by the proton bunch wake, the amplitude of saturated instability. For this, we choose again the wake as a frame of reference moving with the speed ω/k with respect to the laboratory frame of reference. In this case, the speed of bunch protons in this frame (cmpr. with (8)) will be

$$u_1 = -\frac{u\Im m \delta_1 \gamma^2}{1 - \frac{2u^2}{c^2}\Im m \delta_1 \gamma^2} = -\frac{u\Im m \delta_1 \gamma^2}{1 - 2\Im m \delta_1(\gamma^2 - 1)}. \qquad (17)$$

From Eq. (17) we get the following result: in the relativistic limit when $(\delta\gamma^2 << 1)$ this speed is small $u_1 \simeq -u\delta_1\gamma^2 << u$, whereas in the ultrarelativistic limit $(\delta\gamma^2 >> 1)$ the speed is high $u_1 \simeq u/2$. It is worth to point out that ultrarelativity of the proton bunch can be reached at much higher proton energies : γ^2 should be at least one order of magnitude higher compared to that of the electron bunch. For example, when for electron bunch - $n_b/n_p \sim 10^{-3}$, the ultrarelativity can be reached at $\gamma \geq 10$ (electron bunch energy is higher than 5 MeV), whereas for the proton bunch at the same density ratio - only at $\gamma \geq 50$ (proton bunch energy should be higher than 44 GeV).

By analogy with Eq. (9), the stationary saturation amplitude of the plasma wake potential, produced by the proton bunch can be obtained as following

$$\frac{e\Phi_0}{Mc^2} = \frac{1}{\gamma}\left\{\frac{1}{\sqrt{1 - \frac{u^2\delta_1^2\gamma^4}{c^2(1-2\delta_1(\gamma^2-1))^2}}} - 1\right\}, \tag{18}$$

here $\Phi_0\gamma$ is measured in the wake frame of reference. Correspondingly, in the ultrarelativistic limit when $\delta_1\gamma^2 >> 1$ we get an estimate similar to the Eq. (10)

$$\frac{e\Phi_0}{Mc^2} \approx \frac{0.154}{\gamma}, \tag{19}$$

here Φ_0 is measured in the laboratory frame of reference. Eq. (19) leads the following conclusion: the saturation amplitude of the potential of the plasma wake generated by the ultrarelativistic proton bunch is also dependent on the relativity factor $\Phi_0 = \frac{0.154Mc^2}{e\gamma}$. This can be observed in the Figure 2. As we can see it posesses a maximum. We have estimated it for dense plasma at $n_p = 10^{16}cm^{-3}$, $\gamma_0 \simeq 289$ leading to $\Phi_{0max} \simeq 1.1 \cdot 10^5$ V $\simeq 0.1$ M V and rare plasma at $n_p = 6 \cdot 10^{12}cm^{-3}$, $\gamma_0 \simeq 11.6$ leading to $\Phi_{0max} \simeq 5.74 \cdot 10^6$ V $\simeq 5.74$ M V, see Appendix A.

Again, it is quite easy to estimate the amplitude of the electrial field of the wake generated by the proton bunch. For this in Eq. (18), taking into account that $E_0 \sim k\Phi_0$, we obtain the following equation for the relative energy density of the electrostatic field

$$\frac{E_0^2}{8\pi n_p Mc^2\gamma} \simeq \frac{c^2}{u^2}\frac{1}{2\gamma^3}\left\{\frac{1}{\sqrt{1 - \frac{u^2\delta_1^2\gamma^4}{c^2(1-2\delta_1(\gamma^2-1))^2}}} - 1\right\}^2 \tag{20}$$

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leading in the ultrarelativistic limit for the proton bunch when $\delta_1 \gamma^2 >> 1$ to (cmpr. with (12))

$$\frac{E_0^2}{8\pi n_p M c^2 \gamma} \simeq \frac{0.012}{\gamma^3}. \tag{21}$$

We can readily see that the amplitude of the potential as well as that of the electric field of the wake generated by the proton bunch in its turn is defined by the relativity factor as it is shown in the Figure 2 and plasma density n_p, namely $E_0 = 0.024\sqrt{4\pi n_p M c^2}/\gamma$. For ex., for dense plasma at $n_p = 10^{16} cm^{-3}$, $\gamma_0 \simeq 289$ we have $E_{0max} \simeq 5 \cdot 10^5$ V/cm $\simeq 50$ M V/m, for rare plasma at $n_p = 6 \cdot 10^{12} cm^{-3}$, $\gamma_0 \simeq 11.6$ we have $E_{0max} \simeq 6.3 \cdot 10^5$ V/cm $\simeq 63$ M V/m .

4. Results and Discussions

On the basis of the conducted analysis of the resonance Cherenkov interaction of the electron as well as proton bunches with plasma, we can make the following conclusion: the wake amplitude growth produced by the bunches gets saturated with an increase of bunch energy at a quite high level. The highest amplitude of the electric wakefield is produced by the electron bunch in dense plasma at $n_p = 10^{16} cm^{-3}$ and is of order 14.5 M V/m, whereas that produced by the proton bunch is the highest in the rare plasma at $n_p = 6 \cdot 10^{12} cm^{-3}$ and is of order 63 M V/m. These magnitudes are less than those gained with the help of contemporary quite powerful pulse lasers (10^{15} W/cm^2).

In conclusion it must be noticed that nonlinear plasma Lengmuir wave is unstable and, as it was shown in [10], it breaks down when

$$e\Phi_0 > \{M, m\}c^2\gamma_w,$$

where $\gamma_w = 1/\sqrt{1 - u_{ph}^2/c^2}$ and u_{ph} is the wave phase velocity. In our case $u_{ph} = u$ and $\gamma_w = \gamma$. From the results of our calculations follows that considered resonance Cherenkov instabilities arise as for electron as well as for proton beams when the plasma waves amplitudes are much less than the breakdown threshholds.
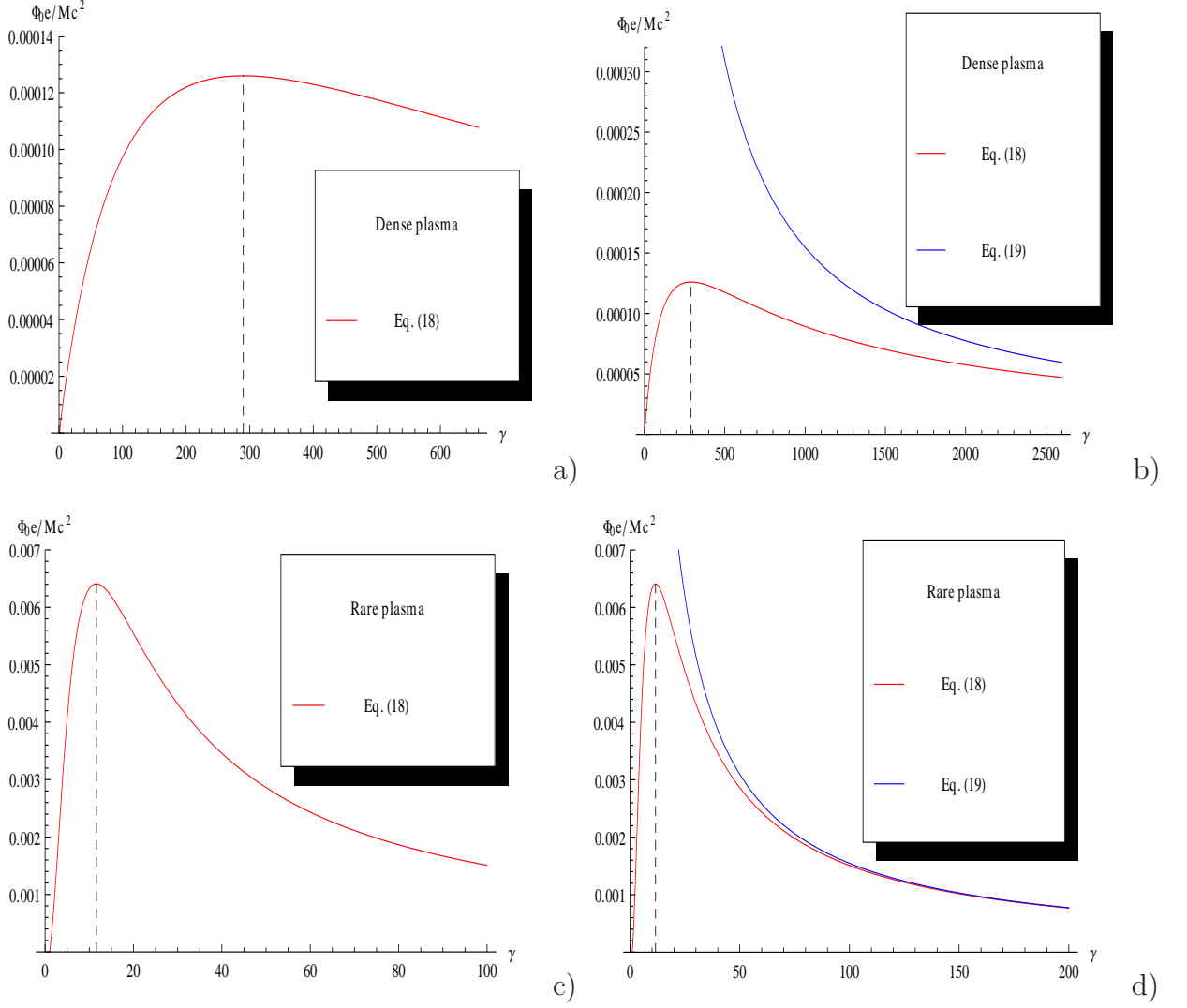
Figure 2: a) The relative stationary saturation amplitude of the plasma wake potential generated by the proton bunch Eq. (18) for a), b) dense (14) at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 10^{16} cm^{-3}$, $\gamma_0 = 289$ and c), d) rare (15) plasmas at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 6 \cdot 10^{12} cm^{-3}$, $\gamma_0 = 11.6$. In b) and d) the relative stationary saturation amplitude in the ultrarelativistic limit (19) is presented for comparison.

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Appendix A. Estimation of the maximum value of the wake field potential

In order to determine the maximum value of the Φ_0 let us represent the Eq. (9) in the following independent on the type of interaction either electron-plasma or proton-plasma:

$$\frac{e\Phi_0}{mc^2} = (\frac{\alpha}{Z})^{1/n} \left\{ \frac{1+2Z}{\sqrt{1+4Z+3Z^2}} - 1 \right\}, \tag{A.1}$$

where $Z = \delta_{\{\},1}\gamma^2 = -\alpha\gamma^n$, $n = 1$ corresponds to the dense plasma and $n = 5/3$ - the rare plasma, $\alpha = \frac{-1}{2}\left(\frac{n_{bi}}{2n_p}\frac{m}{M}\right)^{1/3}$ for proton bunch and $\alpha = \frac{-1}{2}\left(\frac{n_b}{2n_p}\right)^{1/3}$ for electron bunch.

This Eq. (A.1) has a peak at $Z = 0.55546$ for dense plasma (n=1) and the parameters α and γ should be changed according to the realtion

$$0.55546 = \alpha\gamma. \tag{A.2}$$

For rare plasma (n=5/3) at $Z = 1.35695$ we have

$$1.35695 = \alpha\gamma^{5/3}. \tag{A.3}$$

These values correspond to the maxima for dense plasma

$$\frac{e\Phi_0}{\{M,m\}c^2} = -0.06576\alpha \tag{A.4}$$

and for rare plasma

$$\frac{e\Phi_0}{\{M,m\}c^2} = -0.0618\alpha^{3/5}. \tag{A.5}$$

From Eqs. (A.4) and (A.5) follows that we should increase α in such a way that the constraints (4), (5), (6) and (14), (15), $\delta_{\{\},1} \ll 1$, $\gamma^2 \gg 1$ are not violated. As one can readily see the highest maximum saturation amplitude of the plasma wake potential corresponds to the rare plasma, whereas the highest maximum electric wakefield is in addition defined by the plasma density.

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